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ABSTRACT

This is the fifth in a series of publications to help teachers use a mathematics laboratory approach in the elementary school. It provides 64 activities, developed by teachers and field-tested in classrooms. It is designed to be used with any basic mathematics program, as a stimulant to encourage teachers to use their imagination to develop further activities. In developing a set of task cards, the teacher should adjust vocabulary used in the publication and choose concrete materials in terms of knowledge of the particular children and the type and amount of manipulative materials available. For each activity, materials needed, purpose, procedures and suggestions are given. (MNS)

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SUGGESTIONS FOR TEACHING MATHEMATICS USING LABORATORY APPROACHES

EXPERIMENTAL EDITION

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The University of the State of New York
THE STATE EDUCATION DEPARTMENT
Bureau of Elementary Curriculum Development
Albany, New York 12234
1975

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PREFACE

Substantial financial aid to local educational agencies for children of low income families was provided by the Elementary and Secondary Education Act of 1965. Participating school districts have developed a variety of new educational programs to assist children with special educational needs. These programs are based upon local needs assessment with major parental involvement.

The three major priorities for compensatory education programs are bilingual education, reading, and mathematics. One of the highest priority programs under ESEA Title I is the subject matter area of mathematics. Experience has shown that children who have experienced difficulty learning in a traditional program often react with enthusiasm to a mathematics laboratory approach. This publication was developed to provide practical application of this mathematics approach for use by classroom teachers. It should provide practical suggestions for teachers working directly with educationally disadvantaged children.

Irving Ratchick
Assistant Commissioner for Compensatory Education

FOREWORD

The Bureau of Elementary Curriculum Development and the Bureau of Mathematics Education in cooperation with the Division of Education for the Disadvantaged, ESEA Title I, have developed a variety of materials on the use of a Mathematics Laboratory approach on the elementary level. This joint effort has resulted in the release of the following publications:

Operating A Math Lab, which gives a short overview of the method.

ESEA Title I, Anatomy of an Elementary Project, which gives a concrete example of the use of a Mathematics Laboratory approach with children with identified weaknesses in the mathematics area of problem solving.

Suggestions for Teaching Mathematics Using Laboratory Approaches Grades 1-6

1. Number and Numeration
2. Operations
3. Geometry
4. Measurement

These publications provide practical suggestions for teachers who wish to work with their students within the confines of a laboratory approach.

Encouraged by the response of teachers and administrators to these publications, a decision was made to move even further in the direction of providing concrete activities for teachers who wish to move into the humanistic approach inherent in a Mathematics Laboratory program.

Fredric Paul of the Bureau of Mathematics Education and Peter A. Martin of the Bureau of Elementary Curriculum Development began the task of

developing more advanced activities for teacher use. A committee of experienced teachers consisting of Edward Cunningham, Simone Katz, and Sylvia Orans, under the direction of Elaine Mintz, was engaged as a writing team. Mrs. Mintz is director of elementary mathematics and the other three are classroom teachers. The material completed by this team was field tested in the classroom. As a result of favorable reactions additional experimental activities have been produced for use by school districts.

This publication is the fifth which has been developed for teacher use. Each incorporates the latest thinking of the mathematics revision committee and may be utilized with any basic mathematics program. The publication is designed to serve as a stimulant to encourage teachers to employ their imaginations to develop further activities. The classroom teacher in developing a set of "task cards" will adjust vocabulary and choose concrete materials in terms of an intimate knowledge of the ability levels of the children involved and the type and amount of manipulative materials available. Suggestions are welcome and should be sent to Fredric Paul, Bureau of Mathematics Education, State Education Department, Albany, New York 12234.

Peter A. Martin of the Bureau of Elementary Curriculum Development did the final editing, contributed material and illustrations, and prepared the materials for publication.

Robert H. Johnstone, Chief
Bureau of Elementary Curriculum
Development

Gordon E. Van Hooft, Director
Division of Curriculum Development

TEACHING MATHEMATICS USING LABORATORY APPROACHES

5-1 The Lever.

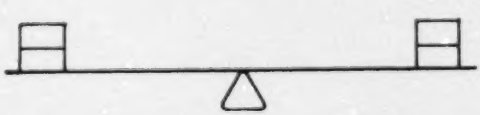
Materials: Balance beam with movable fulcrum, blocks of equal weight, tape measure or ruler.

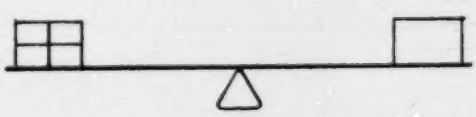
Purpose: Formulating the law of the lever, whole number and fraction computation.

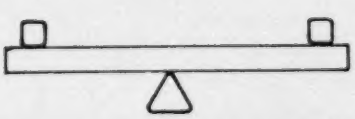
Procedure: Start with a beam whose fulcrum is centered. Have the student balance the beam by using one block near the end of each arm. He should then move one block closer to the fulcrum and investigate how to reestablish balance. Pose the problems, "At how many points on a balance beam can two blocks of equal weight balance each other? How could you describe the balance points? How could you describe the balance relationships, mathematically in an equation?"

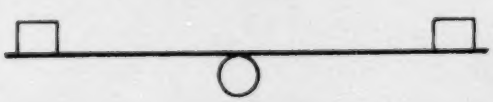
Variations for further experiments -

1. Vary the number of blocks to be balanced.
2. Vary the weight of the blocks to be balanced.
3. Vary the weight and length of the balance beam.
4. Vary the shape of the fulcrum.









5-2 Faces, Vertices, Edges.

Materials: Geometric solid models.

Objectives: Properties of geometric solids, pattern discovery, generalizations.

Procedure: Have students examine various geometric models and make chart as shown. After many models have been viewed, Euler's formula may be observed:

$$F + V = E + 2$$

SHAPES	F Faces	V Vertices	E Edges
CUBE	6	8	12
TETRAHEDRON	4	4	6
ETC.			

Have students investigate whether the formula applies to curved surfaces such as spheres, cones, etc.

5-3 What Does Half of a Rectangle Look Like?

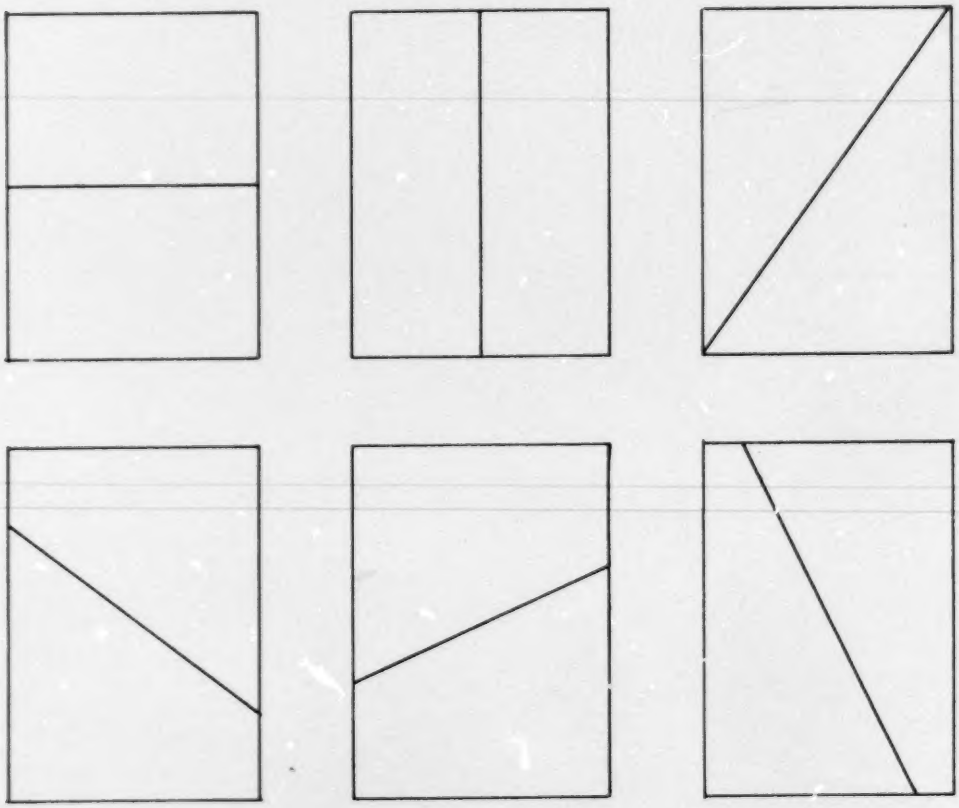
Purpose: Linear and area measurement, congruent shapes, angles, perimeter, fractions, symmetry.

Materials: 6 congruent paper rectangles for each student, scissors, ruler.

Procedure: Student is to find one half of a sheet of paper by a single fold - in six different ways. Cut each paper along the fold and verify that two congruent pieces were made.

Further investigation:

1. Compare the perimeters of each of the six halves.
2. Fold and cut along a diagonal fold and use the two halves to make as many geometrical shapes as possible.
3. Investigate the properties of each shape.



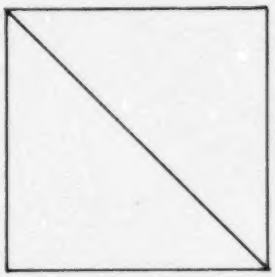
5-4 The Better Half #1.

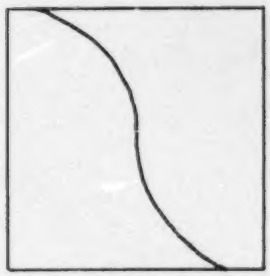
Materials: Paper squares, ruler, compass, scissors.

Purpose: Congruence, symmetry, area, spatial relations, fractions.

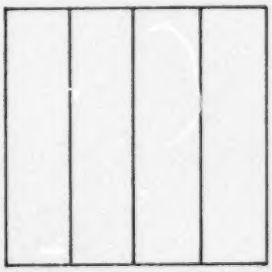
Procedure: Have pupils find out how many ways they can divide a square into two congruent parts, by drawing lines, cutting along these lines, and matching parts.

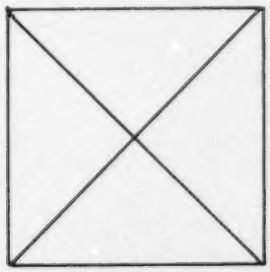
e.g.



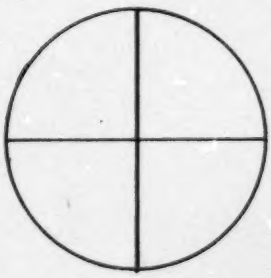


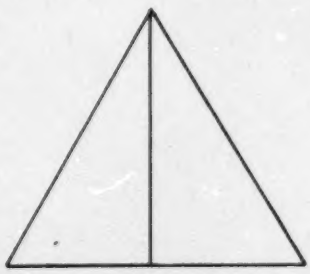
Have pupils find how many different ways they can divide a square into four congruent parts.





Variation: Try the same experiments with rectangles, circles, and regular polygons.

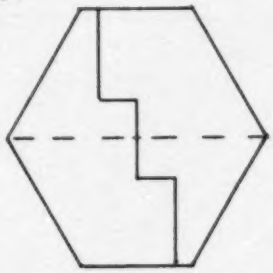




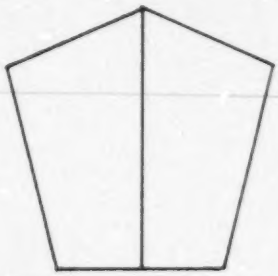
5-4 (Continued)

Note: There is an infinite number of ways to split a regular polygon (with an even number of sides) into two congruent pieces with a single, albeit broken, cut.

As shown in the diagram, cut from one side of the polygon to its center. Then, from the center (the point of symmetry) continue to cut a symmetric pattern to the opposite edge.



Regular polygons with an odd number of sides have line symmetry but not point symmetry. In these cases, half-splitting methods are more restricted.



5-5 The Better Half #2.

Materials: Clay, blunt cutting tool, ruler, protractor, equal pan balance.

Purpose: Congruence, symmetry of point, line and plane, spatial relations, fractions.

Procedure: Have pupils form regular polyhedrons with clay. Using the cutting tool, have pupils find how many different ways they can divide the polyhedron into congruent halves (or quarters.)

Note: This experiment extends the previous planar activity into another dimension and offers greater challenges in the exploration of symmetrical relations.

5-6 Reflections.

Materials: Pencil, paper, ruler, mirror

Objectives: Symmetry, congruence, spatial relations

Procedure: Have pupils draw several geometric figures. Now have them imagine how the figure would look if a mirror were placed on a specified line of the figure. Have them draw the reflection envisioned. Then, they can use mirror to check their work.

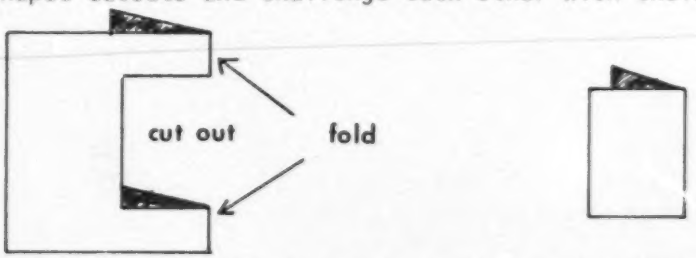
5-7 Unfold and Behold.

Materials: Paper, scissors.

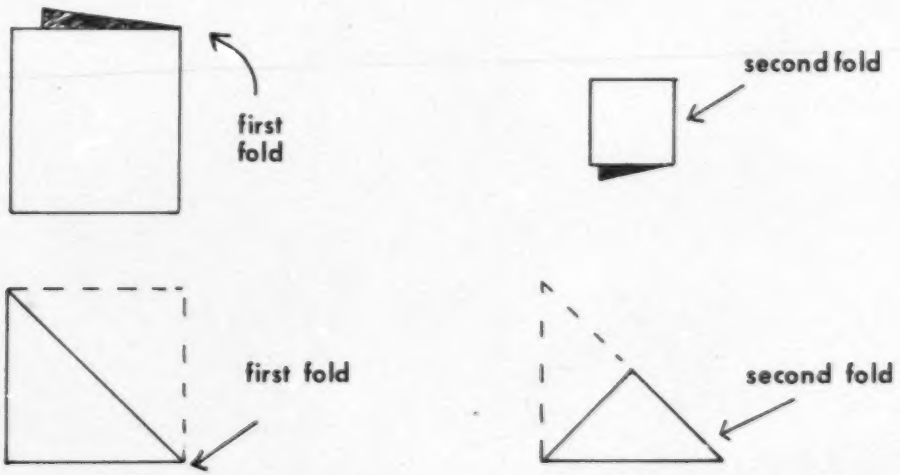
Purpose: Symmetry, polygon properties.

Procedure: The teacher should demonstrate an example of the type of puzzle described below:

Fold a paper once. On the fold, cut out a square. Ask pupils what shape the cutout will have when it is unfolded. Pupils should experiment with different shaped cutouts and challenge each other with their puzzles.



Further variations can be investigated by folding the paper twice (or more times) and then venturing guesses about the shape of cutouts.



5-8 Geoboard Beginnings.

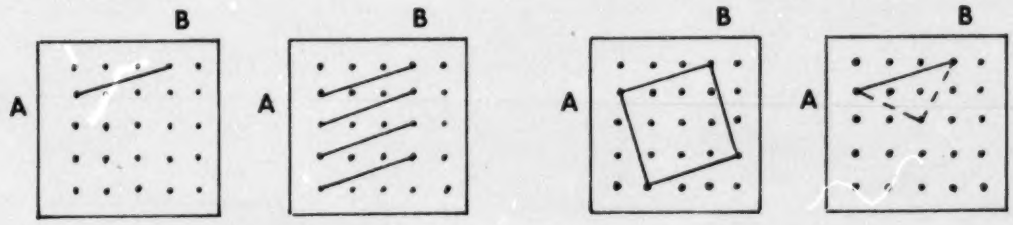
Materials: Geoboards, rubber bands, dot paper or graph paper.

Objectives: To show or draw line segments, parallel and intersecting segments, angles, plane geometric shapes, congruence.

Procedure: On the geoboard, ask children to:

1. Find as many segments as possible that are congruent to $\overline{AB}$.
2. Find as many segments as possible that are not congruent to $\overline{AB}$.
3. Find some segments that are parallel to $\overline{AB}$; that intersect $\overline{AB}$.
4. Find some angles that use either $\overrightarrow{AB}$ or $\overleftarrow{AB}$ as a ray.
5. Form the following shapes using $\overline{AB}$ as a side: triangle, quadrilateral, square rectangle, parallelogram, pentagon.

Have children record their findings and comments on graph paper or dot paper.

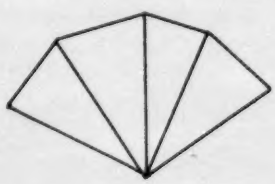


5-9 To A Degree.

Materials: Pencil, paper, ruler.

Purpose: To find the number of degrees in the angles of any polygon; multiplication.

Procedure: Based on information found in the previous experiments, have students analyze the composition of the angles of a polygon. In the example shown, the angles of the four triangles make up the angles of the polygon. Since the pupils have already found that the angle sum of a triangle is 180°, they can multiply to find the polygon angle sum.



5-10 Polygons and Triangles.

Materials: Pencil, paper, ruler.

Purposes: To find through data tabulation: What is greatest number of diagonals that can be drawn from one vertex of a polygon to the remaining vertices? How many triangles are created in the polygon after these diagonals have been drawn?

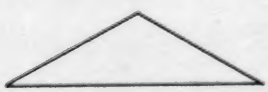
Procedure: Have students draw several assorted polygons. In each, have them draw as many diagonals as possible from a single vertex. Devise a table for recording data (as shown below).

Polygon

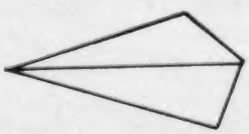
No. of sides	Name	No. of diagonals	No. of triangles
3	Triangle	0	1
4	Quadrilateral	1	2
5	Pentagon	2	3
etc.	. . . .	. . . .	. . . .
n	n-gon	n-3	n-2

Encourage pupils to discover the general relationship--that the number of triangles is two less than the number of sides of the polygon.

a) Triangle

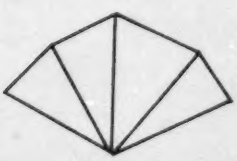


b) Quadrilateral

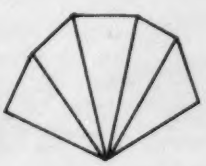


c) Pentagon

d) Hexagon



e) Heptagon



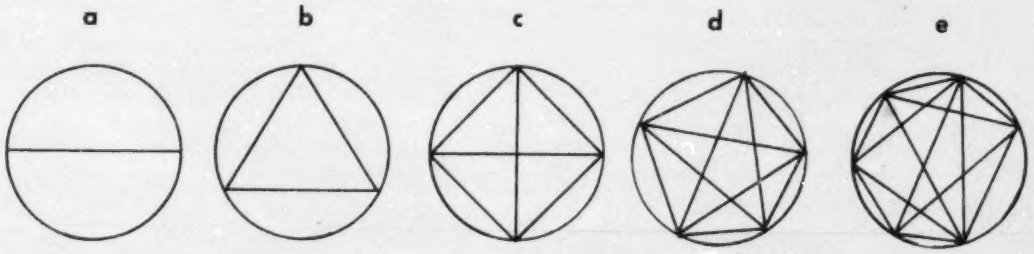
5-11 The Right Connections.

Materials: Paper, compass, ruler, protractor.

Purposes: Geometric designs and patterns, spatial relations, data tabulation, generalizations.

Procedure: Have student draw nine independent circles. He is then to mark each circle with variously spaced points as follows: First circle - 2 points; second circle - 3 points; third circle - 4 points; .....; ninth circle - 10 points. Within each circle, the student should be asked to connect all possible pairs of marked points with a straight line segment. He should keep a record of his findings as indicated below.

What predictions can be made for the number of connecting lines possible on circles with more marked points?



Circle	No. of marked points	No. of lines possible
a	2	1
b	3	3
c	4	6
d	5	10
e	6	15
f etc.	?	?

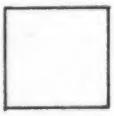
Differences between possible lines
b-a=2
c-b=3
d-c=4
e-d=5
f-e=2

5-12 Building Squares

Materials: Unit squares, blocks, or graph paper.

Purpose: Perimeter, area, integration of arithmetic and geometric concepts, sequential pattern, numeration, exponents, multiplication.

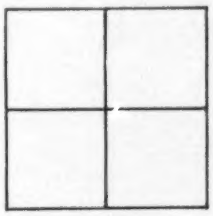
Procedure: Have pupils assemble squares based on a given module, so that the side of each successive square is increased by one unit. Have pupils investigate properties of the squares they have built; for example:



1

1×1

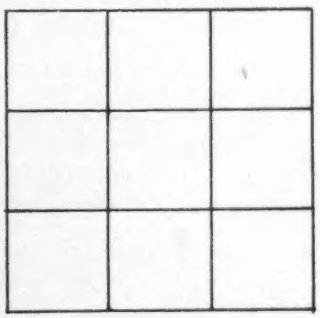
1^2



4

2×2

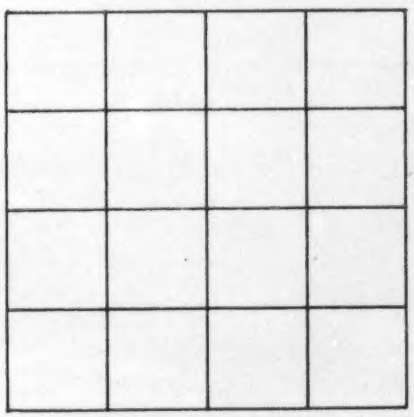
2^2



9

3×3

3^2



16

4×4

4^2

Suggestions:

1. The squares alternate (odd, even).
2. A table of the differences between adjacent squares produces the sequence of odd numbers.
3. The perimeters increase at a constant rate of 4 units while the areas increase in an odd number pattern.

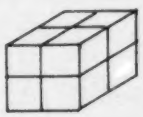
5-13 Building Cubes.

Materials: Wooden or plastic cubes.

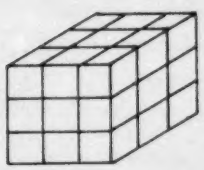
Purpose: Perimeter, area, volume; integration of arithmetic and geometric concepts; sequential patterns; numeration; exponents, multiplication.

Procedure: Have pupils assemble cubes based on the given module, so that the edge of each successive cube is increased by one unit. Have pupils investigate properties of the cubes they have built. Compare this project with the previous one.

1
$1 \times 1 \times 1$
1^3



8
$2 \times 2 \times 2$
2^3



27
$3 \times 3 \times 3$
3^3

Suggestions:

1. The total number of unit cubes alternate (odd, even).
2. The sum of the distances along the edges of each cube is a multiple of 12.

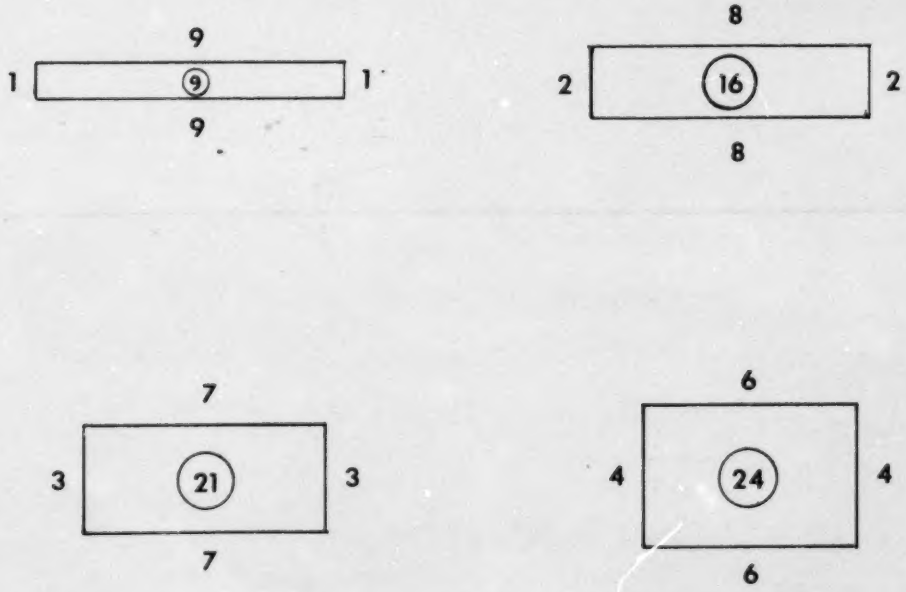
5-14 Corralling the Cubes - Constant Perimeter.

Materials: Strings of different lengths (even-number measures, e.g., 20 inches, 38 centimeters, etc.), and set of unit blocks or graph paper.

Purposes: To discover a perimeter formula for rectangles; data tabulation and analysis; rectangle properties.

Procedure: Have pupils arrange the string to form a rectangular "fence." Have them estimate the number of cubes they think will fill the corral. The estimate should then be verified. Have pupils investigate whether different rectangular "corrals" could be made with the same string "fence."

Example: 20-inch string



Questions:

1. What is observed about pairs of opposite sides? Pairs of adjacent sides?
2. How does a change in one side affect the others?
3. Is there a corral with a minimum area for the 20-inch string? A maximum area corral?

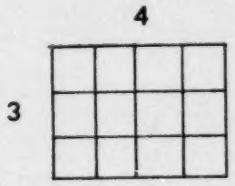
5-15 Corralling the Cubes - Constant Area.

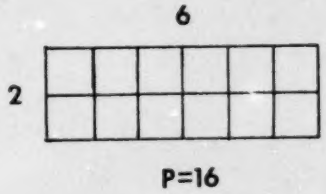
Materials: Cubes, squared material, or geoboard.

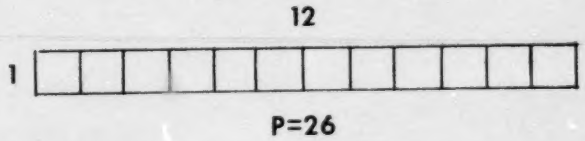
Purpose: Area, perimeter; multiplication, factors, data tabulation.

Procedure: Have pupils choose a specific number of cubes or squares to serve as their constant area. Using each of the cubes, pupils should build as many different rectangular arrays as possible. They should then record findings and make comparisons. Investigate: which rectangle has the greatest perimeter? the smallest perimeter?

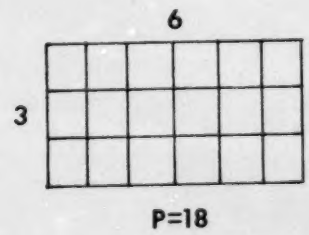
Example: Rectangular arrays with an area of 12.

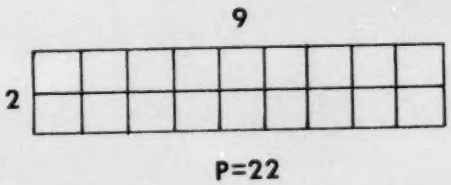


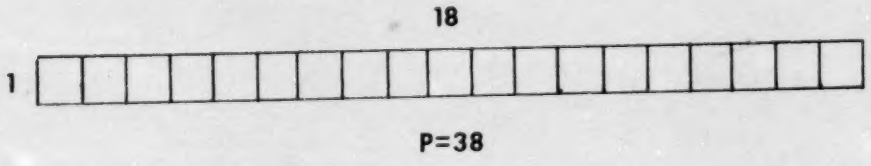




Example: Rectangular arrays with an area of 18.







5-16 Corralling the Cubes - Constant Volume.

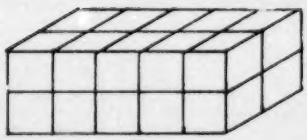
Materials: Cubes

Objectives: Multiplication, volume, geometric shapes.

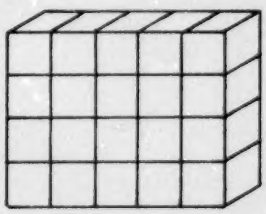
Procedure: Have pupils choose a specific number of cubes to serve as their constant volume. Using each of these cubes, have pupils build as many rectangular solids as possible. They can record their findings on a chart.

For example:

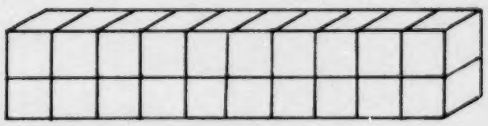
Volume of 20 cubes.



5×2×2



5×4×1



10×2×1

Questions:

Is there a rectangular solid having 20 cubes that has a) the smallest number of squares on its face? b) largest number of squares on its face? c) the smallest surface area? d) the largest surface area?

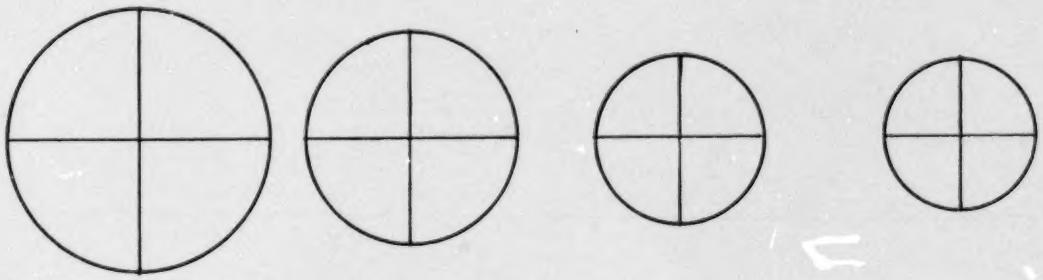
5-17 Squares for the Circle.

Materials: Graph paper, compass.

Purpose: To find the area of a circle experimentally by direct count; data tabulation, generalizations.

Procedure: Have the student use a compass to draw a circle with its center on a lattice point of the graph paper. Let the radius length be a whole number. By drawing two perpendicular diameters, the student can divide the circle into quarters. He can approximate the circle's area by counting the squares enclosed in a quarter of the circle and multiplying by four.

The experiment should be repeated several times with circles of different radii. The student should record his observations and report when he recognizes a pattern emerging. (The area should approximate three times the radius.)



5-18 The Consistent Circle.

Purpose: Experimental discovery and verification of the formula: Circumference $\approx$ 3 x diameter; approximating π, ratio, data tabulation, graphing.

Materials: Several objects with circular cross-sections such as cylindrical cans, cups, waste paper baskets, jar covers, etc., tape measure.

Procedure: Have pupils measure the diameter and circumference of each object and record their data on a chart and/or a graph. When pupils compare the sizes, they should note the constant ratio between the diameter and circumference in each case. Elicit the formula, $C = \pi d$, which generalizes the relationship.

Object	Diameter	Circumference
A		
B		
C		
...		

5-19 Circle Area - Formula and Graph.

Purpose: Circle properties interpreted and reinforced graphically.

Materials: Circular objects, measuring instruments, graph paper.

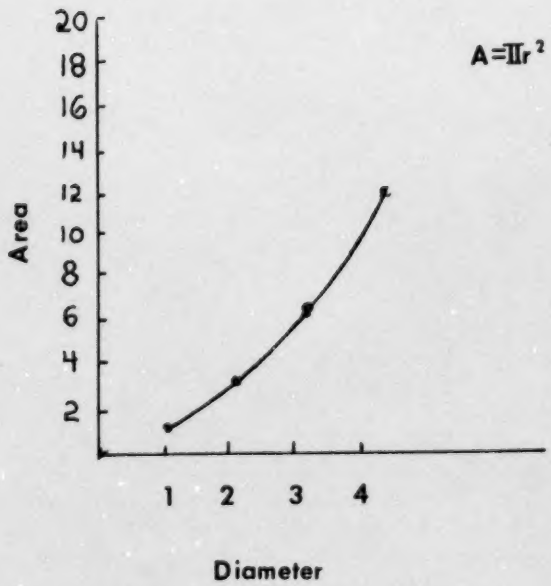
Procedure: After developing the formula, $A = \pi r^2$, have pupil measure the diameters of various circular objects. Have them apply the formula to prepare a data chart for a graph.

Problems to investigate:

Is the graph straight? Curved? Given a circle's diameter, how can its area be estimated from the graph? If the area of a circle is known, can its diameter be found from the graph?

Note: Comparable exercises can be developed with the perimeter and area formulas for other geometric shapes.

Object	Diameter	Area
A		
B		
C		
...		

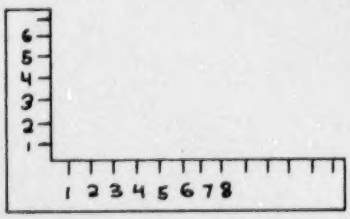


5-20 Making and Using an Area Ruler.

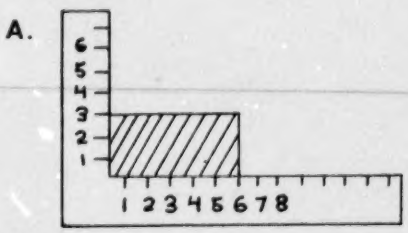
Materials: Cardboard, ruler, protractor, scissors, marker.

Objectives: Making and using a measuring device to find the area of rectangles and triangles; area formulas reinforced, linear measurement, multiplication.

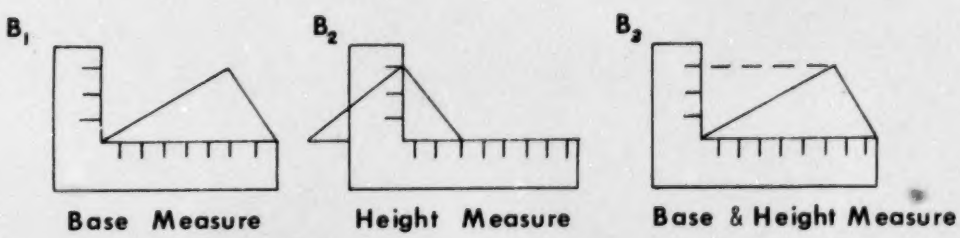
Procedure: Have students draw a right angle approximately 2 inches wide and as long as desired. It can be ruled with inch or metric spacing, or both.



A. To calculate the area of a rectangle, position the device, as shown in diagram A, to give both dimensions of the rectangle. Then, apply the formula for the area of a rectangle.



B. To calculate the area of a triangle, position the device as shown in diagrams B_1 and B_2 or B_3. Then apply the formula for the area of a triangle.



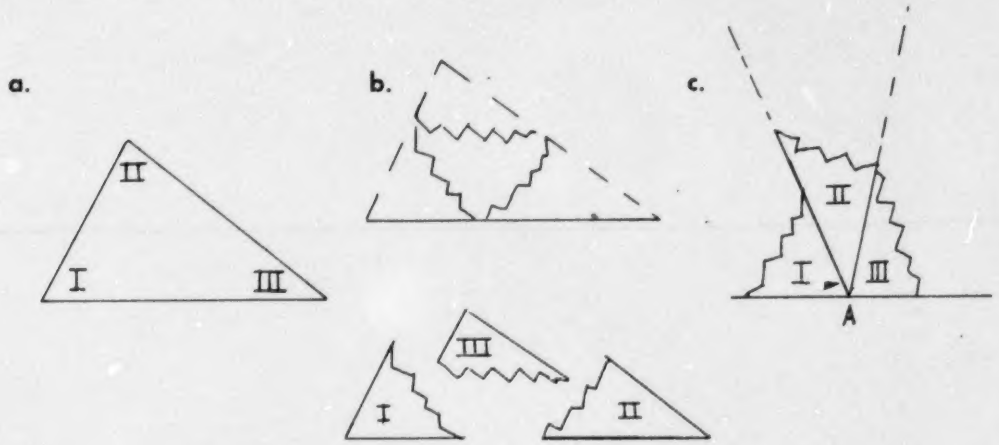
5-21 The Angle Sum for Any Triangle.

Materials: Paper, pencil, ruler, scissors, paste or tape.

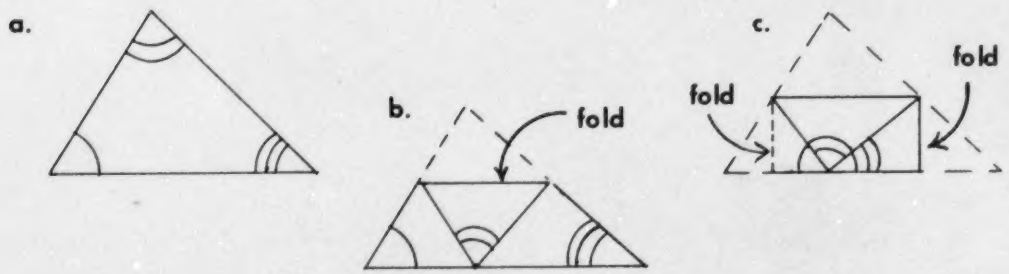
Purpose: Experimentally verify that the sum of the angles of any triangle is a straight angle; triangle classification by sides and/or angles.

Procedure: Have the student:

1. Draw and cut out a triangle.
2. Carefully tear each angle from the triangle.
3. On a separate sheet of paper, rule a straight line and mark a point, "A," which will serve as a common vertex.
4. Arrange the three angles at point "A" with no overlapping (as shown in diagram).
5. Repeat steps 1-4 with a variety of triangles - scalene, isosceles, equilateral, acute, right, obtuse.
6. Note that in each case, the three angles of any triangle may be arranged as a straight angle.



Alternate Approach



Fold triangle so the three vertices meet at one point.

5-22 Getting in Shape.

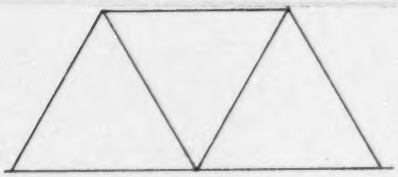
Materials: Assorted Plane Geometric Shapes.

Objectives: Shape fitting, angle recognition and measure, regular and irregular shapes, congruence, polygon properties, concave, convex, symmetry.

Procedure: A regular shape is equilateral and equiangular. Have pupils find what regular and non-regular shapes can be made by placing the following groups of shapes on a plane so that adjacent figures share a common side.

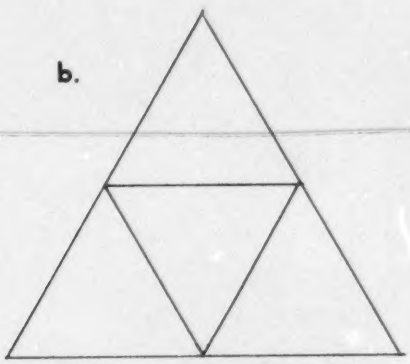
a) three congruent equilateral triangles
b) four " " "
c) six " " "
d) two " right-angled "
e) four " " " "
f) six " isosceles "
g) four " squares " etc.

a.



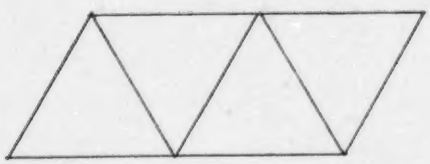
Trapezoid

b.

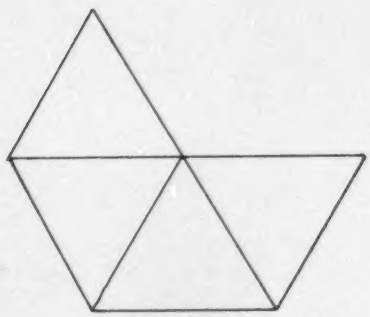


Triangle

ect.



Parallelogram



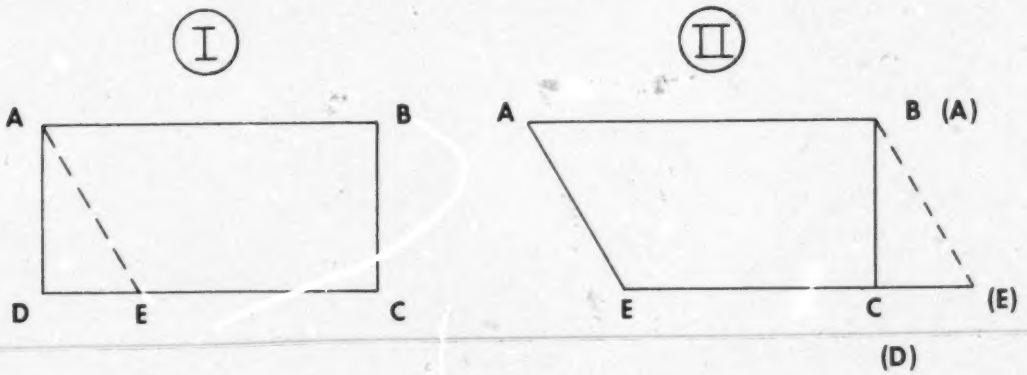
Concave Hexagon

5-23 Space Travel and the Parallelogram.

Materials: Paper rectangle (if desired, use graph paper), rulers, scissors.

Purposes: Area of parallelogram, area conservation, quadrilateral properties.

Procedure: Have the student measure the length and width of his rectangle A B C D and find its area. Next, have him draw a line from vertex A to any point E on the opposite side $\overline{CD}$. Cut segment AE and place ADE so that $\overline{AD}$ coincides with BC as shown in diagram II.



Discuss what kind of shape figure II is and what its relation is to figure I.

5-24 Tile the Floor.

Materials: Trundle wheel or measuring tape.

Purpose: Linear and area measurement, estimation, computation, research, economics.

Procedure: 1) Students should estimate how many 12-inch square and/or 9-inch square tiles would be needed to cover the classroom floor.

2) Students should then measure the dimensions of the room, compute the actual area, and compare the results with the original estimates.

3) Research - students can find the number of tiles delivered in a carton and the price range of vinyl and asbestos tiles.

a) How many cartons are needed for the classroom?
b) How much would the tiles cost (with and without sales tax)?
c) How much floor space would be needed to store the tiles if the cartons were set out in a single layer?

5-25 It's a Puzzlement.

Materials: None

Purpose: Recreation, pattern development, exponents, research.

Procedure: Pose the problem: How many different paths can be taken to spell "Math Labs" on the given puzzle board if you start at any "M" and move horizontally or vertically toward the "S"? Answer: 255. (A rationale for solving the puzzle appears below.)

```
                M
              M A M
            M A T A M
          M A T H T A M
        M A T H L H T A M
      M A T H L A B H T A M
    M A T H L A B A L H T A M
  M A T H L A B S B A L H T A M
```

1)
```
  M
M A M
```

2)
```
    M
  M A M
M A T A M
```

3)
```
      M
    M A M
  M A T A M
M A T H T A M
```

4)
```
        M
      M A M
    M A T A M
  M A T H T A M
M A T H L H T A M
```

Paths:

$2^2 - 1 = 3$ $2^3 - 1 = 7$ $2^4 - 1 = 15$ $2^5 - 1 = 31$

Case 1) - To spell "MA," there is only one path from each "M."

Case 2) - To spell "MAT," there is one path from each corner "M" and two paths from each middle "M."

Case 3) - To spell "MATH," there is one path from each corner "M" and three paths from the "M"'s on lines 2 and 3.

Case 4) - To spell "MATHL," there is one path from each corner "M," four paths from each "M" on lines 2 and 4 and six paths from each "M" on line 3.

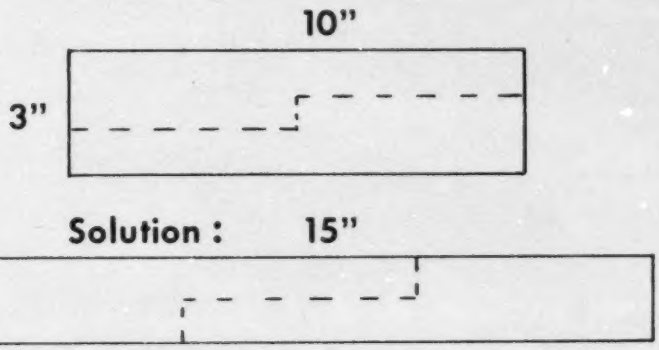
Note: This puzzle is related to the Binomial Expansion and Pascal's Triangle.

5-26 A Tricky Area Puzzler.

Materials: Scissors, unlined paper, graph paper.

Purpose: Measurement, area, perimeter, geometric perception, congruence, conservation.

Procedure: The "puzzler" is: Can you cut a piece of paper 3" x 10" into two congruent pieces so that a new rectangle with dimensions of 2" x 15" can be formed by joining the pieces? Note the area remains constant (conservation) while the perimeter changes. Encourage pupils to do paper folding and cutting before proceding to the use of graph paper.



5-27 Stringing Along Your Foot.

Materials: String, graph paper, scissors, ruler, tape.

Purpose: Linear and area measurement, perimeter, ratio, proportion, scale generalizations.

Procedure: Have each student trace the outline of his foot on graph paper. Using string, students are to measure the perimeters. The area enclosed by the outline is to be calculated by counting square units.

5-28 Gift Package.

Materials: Rectangular boxes with lids, or tissue boxes, contact paper or gift wrapping paper, ruler, scissors, rubber cement, scotch tape, protractor or T-square.

Purpose: Linear, area, and angle measurement, estimation, geometric shapes, computation.

Procedure: Students should estimate the total area of the surfaces of their boxes before actually measuring and calculating the areas. Using geometric tools, the students should measure and cut out precise paper patterns for covering the boxes. Pupils may be challenged to cover their boxes with no more than three pieces of paper. (One piece for four sides, and two additional pieces for the remaining two sides.)

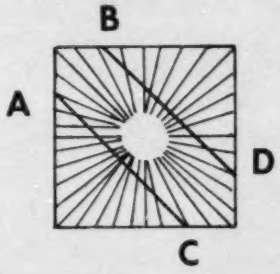
5-29 Optical Illusions.

Materials: Examples of illusions (See samples below, research sources.)

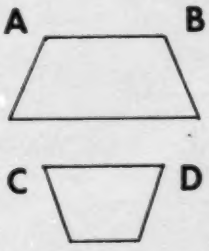
Purpose: Properties of geometric shapes, congruence, parallelism, measurement.

Procedure: Pupils should be encouraged to do research on optical illustions. They can investigate the perceptual, geometric, artistic, and commercial implications of "trompe d'oeuil."

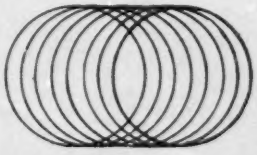
1. Are $\overline{AB}$ and $\overline{CD}$ parallel or bent?



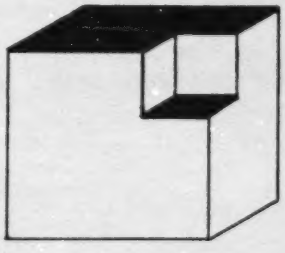
2. How do the sizes of $\overline{AB}$ and $\overline{CD}$ compare?



3. Does the cylinder open on the right or on the left?



4. Can you see the figure in three different ways?



29

5-30 How Much Do Your Feet Bear?

Materials: Graph paper.

Purpose: Area and weight measurement, division, ratio, estimation, data tabulation.

Procedure:
1) Have student trace his foot on graph paper to determine its area in square units. (Count the squares.)
2) Since the body weight rests on both feet, multiply the result by 2.
3) Have student weigh himself or herself.
4) Divide the weight by the total foot area.
5) Each student should record his statistics on a class chart.

Student	Weight	Foot Area	Weight / Foot Area
Mary			
John			
Jim			

6) Does any consistent pattern emerge? (Note: the average ratio of weight: foot area is approximately two pounds per square inch.)

5-31 How Much Skin Covers You?

Materials: Transparent grid, string, unlined paper.

Purpose: Area measurement, ratio, estimation, percentage.

Procedure: Have each student trace the outline of his foot on unlined paper and estimate the area. Place the grid over the outline. Count the squares that fall within the outline for the area measure. Compare this with the estimate.

Note: The area of the skin of the foot is about 1% of the body's entire skin area.

5-32 Your Heartbeat.

Materials: Watch with a second hand.

Purpose: Measurement, time-rate, data tabulation, research.

Procedure: a) To compute the pulse rate per minute, the pulse beat is counted for 30 seconds. This count is multiplied by two.
b) Pupils will work in pairs, alternating roles as pulse taker and pulse giver. They will observe and record pulse changes under various conditions, as suggested below:
c) Various body conditions:
1. Seated
2. Person jumps in place 20 times. Pulse count is taken immediately after the exercise; pulse count is taken at intervals of one minute after the exercise.

Body Condition	PULSE RATE CHART 30-Second Count	Pulse Rate Per Minute
Student Seated		
Teacher Seated		
Student Jump in Place		
Teacher Jump in Place		
Second Student Seated		
Second Student Jump in Place		

e) Students can find out the range of normal pulse rates for children and for adults.

5-33 Can You Find an Average Person?

Materials: Tape measure, scale, graph paper, string.

Purpose: Measurement - linear, area, weight; whole number and fraction operations; proportion; graphing; statistical analysis; estimation.

Procedure: a) Have pupils:
1. discuss the categories of body measures that can be ascertained from head to toe.
2. determine the size and nature of the sample population that they will measure.
3. determine the responsibility of each pupil to collect data that will contribute to the project.
4. devise the format to be used for data. Suggested categories for data: height, weight, span, pace, palms, cubits, thumbs, wrists, necks, sizes for hats, gloves, belts, shoes, rings.

b) For each data category, have pupils:

1. calculate the mean, median, and mode.
2. graph the data.
3. mark the mean, median, and mode of the graph.

c) Display the entire set of graphs. Compare their shapes. Consider the "average" person for each category. Are any people average in all categories? Many categories?

CAN YOU FIND AN AVERAGE PERSON ?

5-34 A Leaf and a Scrap.

Materials: Leaves, fabric scraps of various shapes, graph paper, string.

Purpose: Area measurement, estimation.

Procedure: Students are to choose from the scraps and/or leaves, two different shapes which they believe to have the same approximate area; estimate the number of square units. Graph paper can be used to check the accuracy of the area estimated.

5-35 A Dog's Life.

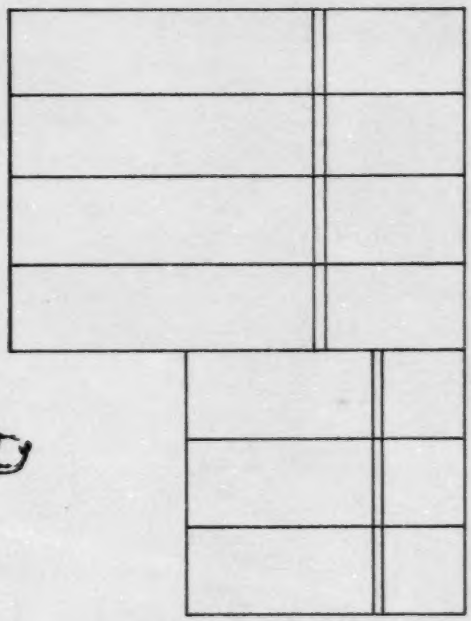
Materials: Ruler, graph paper.

Purpose: Linear and area measurement, scale drawing, data collection.

Procedure: In an interview with a veterinarian, the students should get the following data:

a) 1. What are the dimensions of the smallest kennel space in the kennel? The largest?
   2. How many kennel spaces are there of these sizes?
   3. How many kennel spaces are there altogether?
   4. How many spaces in each size category?
   5. Are spaces used for more than one animal? How many?
   6. How large is one exercise area?
   7. How many exercise areas are there of the same size?

b) Student will make a scale drawing for a kennel for at least 10 dogs.
   1. Make a sketch first to show kennel spaces and exercise areas. Use dimensions from above data. Label and indicate measurements.
   2. Make a scale drawing of the sketch.



5-36 How Much News Is There in a Newspaper?

Materials: Newspaper, ruler.

Purpose: Linear measurement, multiplication, addition, fractions, economics, research.

Procedure: Have students look at an inside page of a newspaper to determine how many columns of print are on one page. Have students measure the length of a full column and find how many column inches are on one page (a column inch is measured vertically), and how many column inches are in the whole paper. Have students list categories, e.g., news advertising, entertainment, and record the number of column inches for each category. Have then approximate the fractional part of the paper each category occupies.

Have students find the cost of newspaper advertising space. Make comparisons between the cost of advertising in various media.

5-37 The Lowly Shadow Rises.

Materials: Yardstick or metal tape measure.

Purpose: Ratio, proportion, data tabulation, graphing.

Procedure: On a sunny day, have student measure various out-of-doors objects and their shadows, e.g., fence, bush, person. He can make a chart, as follows:

Object	Height of Object	Length of Shadow	Ratio H/L
Bush	2 ft.	1 ft.	2/1
Child	4 ft.	2 ft.	4/2
Fence	6 ft.	3 ft.	6/3
Pole	8 ft.	4 ft.	8/4
Tree	10 ft.	5 ft.	10/5

Does each pair of ratios form a proportion? Check by finding cross-products. Can you now find the height of another object by just measuring its shadow? Have pupils graph the results.

5-38 Rocks and Water.

Materials: Four rocks (about size of one's fist), labelled A, B, C, D. Locks or other metal objects, spring scale, pail at least 10 inches high, water, strip of cloth 2"x10".

Purpose: Weight measurement, tabulation, estimation, graphing, ratios, proportions, generalizations.

Procedure: a) Make a sling with a cloth and attach it to the hook of the spring scale.
b) Weigh each rock in the sling and record its weight.
c) Fill pail halfway with water.
d) Tabulate the data as follows:

Objects	Weight in Air	Weight in Water
Rock A	____ units	____ units
Rock B	____ units	____ units
...	...	...

e) Using the sling, suspend each rock in the water.
f) Weigh rocks A, B, and C in water, and record their weights.
g) How can the weight of rock D in water be predicted? Estimate it.
h) Check the estimate for the weight of rock D in water.
i) Choose two objects that weigh the same in air. Find out their weights in water. Are the weights equivalent in water? Explain.

5-39 Watch Out! The Earth Is Shifting!

Materials: Encyclopedia, Almanac, globe.

Purpose: Measurement, introduction of the Richter scale, multiplication, research, exponents, patterns, ratio, geometric progression, energy units.

Procedure: Students should investigate the significance of the Richter scale as applied in measuring earthquakes. The scale values are numbered from 1 to 10 and indicate increases in energy in a geometric progression. From a scale value of 2 which represents 60 times the energy of a scale value of 1, each successive unit in the scale is 60 times the preceding unit.

Have students find out when and where the major earthquakes occurred in the last ten years, and how they rated on the Richter scale.

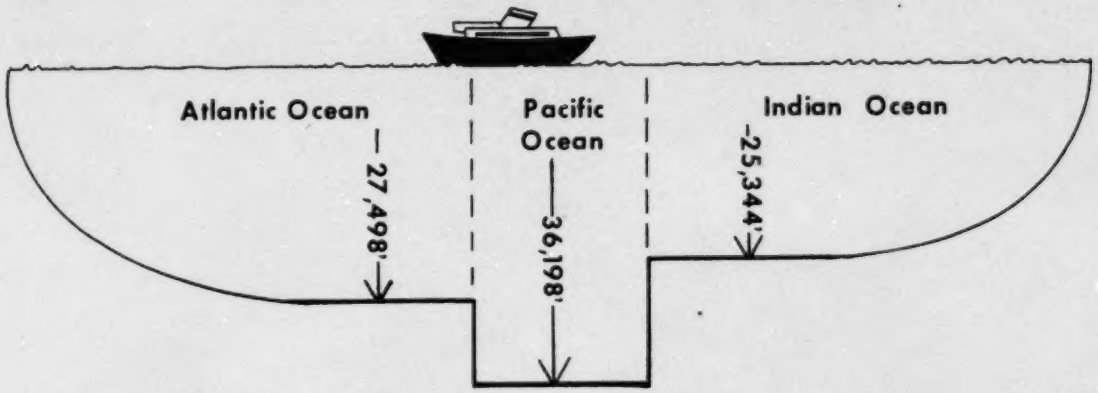
5-40 How Deep Is the Ocean?

Materials: Almanac, encyclopedia.

Purpose: Linear measurement, research, data tabulation, graphing.

Procedure: Have students do research and record the depths of the three deepest points discovered in each of the following oceans: the Atlantic, Pacific and Indian. This data should be organized in a table graphically.

Have students also find out how deep explorers can go in the ocean with and without special gear, and with submarines. What kind of equipment do explorers use to gauge the depth of the oceans?



5-41 A Canal Is a Short Cut.

Materials: String, globe, encyclopedia, ruler, flat world map, map measurer.

Purpose: Linear measure, whole number and fractional operations, map scales, ratio, distance related to time and speed, research.

Procedure:

a) Have students lay string on a globe to find out the distance that a ship would cover from New York City to San Francisco (1) via the Panama Canal (2) going around Cape Horn.

b) Have students determine three different ocean routes from Singapore to Rio de Janeiro, and find out on a flat map, using a map measurer, how the distances vary.

c) Have students find out the range of speeds for ocean freighters and how much time it would take a freighter to travel each of the routes.

d) Have students compare the speeds of steam freighters with those of the "Yankee Clippers."

5-42 A Picture Is Worth a Thousand Words.

Materials: Camera with shutter speeds of 1/30, 1/60, 1/125, and lens openings from f/4 to f/11; black and white film (ASA 100 or 125), seven index cards, felt-tip marker.

Purpose: Linear measure, time measure, combination of ordered pairs, ratio, patterns, data tabulation.

Procedure: a) On each index card, the student is to mark one of the following shutter speeds or lens openings: f/4, f/5.6, f/8, f/11, 1/30, 1/60, 1/125.

b) The student should set his camera to correspond to a shutter speed and lens opening shown on two cards. There are twelve possible shutter-lens combinations (e.g.; f/4, 1/30).

c) From a distance of ten feet, the student should photograph the pairs of cards correlated to his camera settings.

d) After prints have been processed, place pictures in an array as shown in the chart below:

SHUTTER OPENING	LENS OPENING f 4	f 5.6	f 8	f 11
1/30sec.				
1/60sec.				
1/125sec.				
1/200sec.				

e) The students should assess the qualitative differences of the prints.

5-43 Be a Decorator: Place the Furniture in a Room.

Materials: 1/4" graph paper, ruler, scissors, paste.

Purpose: Linear and area measurement, scale drawing, ratio, whole number and fractional computations.

Procedure: Have students:

a) Calculate scale measurements for the floor space required by each piece of furniture, as shown below:

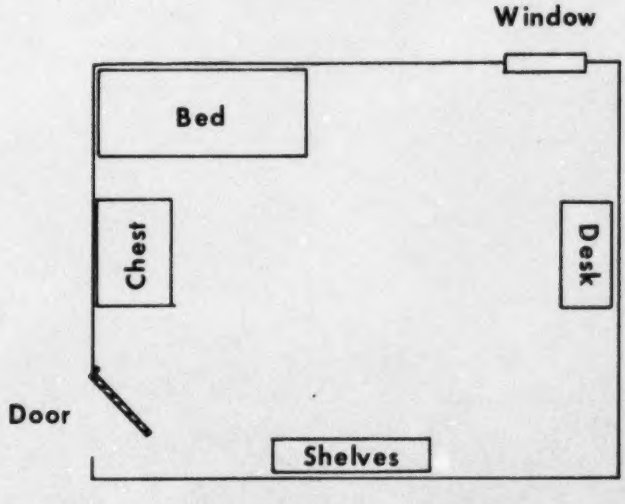
(Recommended scales: 1/2 inch = 1 foot or 1 inch = 1 foot)

1. Room - 12 feet x 15 feet
2. Desk - 1 1/2 feet x 3 feet
3. Chest of drawers - 2 feet x 3 feet
4. Bed - 2 1/3 feet x 6 feet
5. Door - 2 1/2 feet wide
6. Window - 3 1/2 feet wide
7. Bookshelves - 1 foot x 4 feet

b) Draw the room (to scale) on graph paper.

c) Draw, label, and cut out a scale model of each piece of furniture.

d) Arrange the furniture models on the floor plan.



Note: This project may be extended to show a full house design. Also, clay models of furniture can spark the project with a new dimension.

5-44 The Spotlight is on the Wall! (Activity for two students)

Materials: Square centimeter ruled graph paper - about two meters square, (several smaller sheets may be taped together), marker, flashlight, meter stick, masking tape or corkboard and tacks.

Purpose: Linear and area measurement, generalizations, graphing, data tabulation.

Procedure: Students are told to do the following: Choose a wall surface that is fairly dark. Tape or tack graph paper to the wall. Hold flashlight horizontally (do not tilt). Hold flashlight as close to paper as possible, at the center of the paper. As the light shines, have a partner draw around the outline of the light. Determine number of square units (area) of light, and record this way.

Trial	Distance of Flashlight from Paper	Area of Light	Difference in Light Area
A	1cm	_____ sq. cm	
B	10cm	_____ sq. cm	B-A ___ sq. cm
C	20cm	_____ sq. cm	C-B ___ sq. cm
"			

Students should: Predict the increase in the area of light when the flashlight is held 30 cm , 50 cm., and 100 cm away from the wall. The estimates should be tested and the results should be graphed.

Finally, pupils should express a general relationship between the area of illumination and the distance of the flashlight from the paper.

5-45 How Fast Does a Toy Car Go?

Materials: Battery-driven toy car, stopwatch with second hand.

Purpose: Measurement of distance and time, calculation of rate, multiplication, division, ratio, averaging, unit conversion, tabulation of data.

Procedure: Have pupils set up a track. Have them measure the distance between the starting and finishing lines. Pupils are to time the car's journey from start to finish and repeat the experiment at least five times. The data should be recorded.

Pupils should find the average time (in seconds) for the distance (in feet) the car travels. They can compute the rate in feet per second, feet per minute, yards per minute, miles per hour. They can do comparable computations using metric measures.

5-46 Tenting in the Backyard.

Materials: Camping supplies catalog, trundle wheel or yardstick, green carpeting remnants or "astroturf," scissors, suitable materials for making tent models.

Purpose: Linear and area measurement; scale drawing; model making; ratio, whole number, and fractional computations.

Procedure: Have students measure the area of the school yard or some other specified site on school grounds. Using these measures, they can make a scale drawing of the site. They should then determine the size and placement of several tents for the site.

Pupils can make scale models of the tents and calculate the area of the encampment. Calculations can be made of the remaining area. This can be covered with "astroturf."

5-47 What's the Size of It?

Materials: Local map showing sites of places of interest in the community (school, hospital, lake, etc.), acetate grid or tracing paper and graph paper.

Purpose: Linear and area measurement; map reading; scale drawing, ratio, whole number, and fractional computations.

Procedure: Have students use the acetate grid to determine the area covered by the chosen sites, and using the map scale, compute the actual area of the sites.

Alternative to above procedure: Trace the area of sites and transfer this to graph paper. Then, use the map scale to determine area.

5-48 Who Needs a Steeplejack? (Activity for two students)

Materials: Several objects - each two feet or more tall, tape measure, a sunny day. (All activities to be done within 15 minutes, or at same time another day.)

Purpose: Measurement, graphing, generalizations, estimation, ratio, proportion, data tabulation.

Procedure: a) Students place two or more objects in sunny area. Then, the height of each and shadow length of each is measured. A data sheet should be kept as shown below:

	Height	Lenght of Shadow
Object A	_____	_____
Object B	_____	_____
Object C	_____	_____
Object D	_____	_____
Object E	_____	_____

b) Partners measure each other's height and shadow length, and record the data.

c) The shadow length of a flagpole or building is measured and recorded in the proper column.

d) Direct students to find a relationship (1) between the height of each object and its shadow length (2) between the student's height and his shadow length.

e) Pupils should graph the data collected for their investigations.

f) On the basis of the graph and relationships explored in "d," students should be able to estimate the height of the flagpole or building and express the relationship between an object's length and its shadow, in terms of a general equation or formula.

5-49 How Far Is It as the Crow Flies?

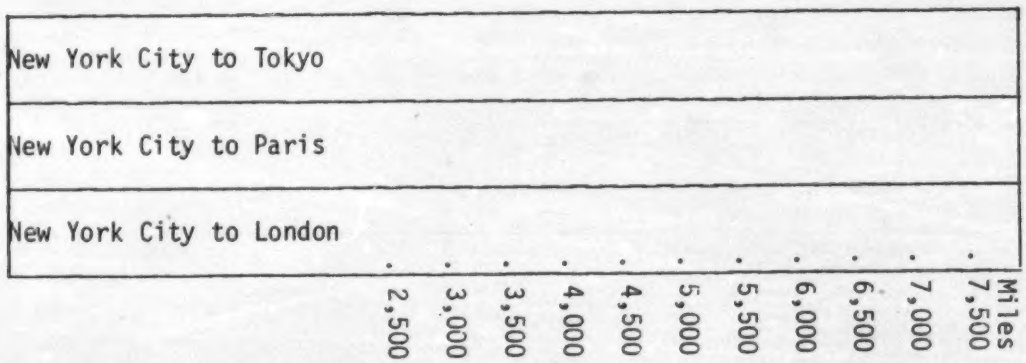
Materials: Globe, string, scissors, graph paper.

Purpose: Linear measurement, data tabulation, graphing, scale, ratio.

Procedure: a) Have children determine the scale ratio used to design their globe and tabulate this information.

Globe Distance (inches)	1/2 in.	3/4 in.	1 in.	2 in.	4 in.
Actual Distance (miles)					

b) Have pupils find how far it is as the crow flies from New York City to London, Paris, Tokyo, etc. One child holds a string taut between two cities on a globe. His partner cuts this length of string. Pupils attach the cut string lengths to a paper to form bar charts.



5-50 How Much Paint to Cover One Wall?

Materials: Tape measure, labels from interior house paint cans, yard and/or meter stick.

Purpose: Linear, area, and liquid measurement, whole number and fractional computations, economics research.

Procedure: The student is to measure the area of a classroom wall, perferably one with windows. Then, he should measure the area of the windows. This is to be subtracted from the wall area to obtain the total area to be painted. The student should find out how much wall area a gallon of paint covers. He should then determine the number of gallons he would need, to cover his wall. Should he expect to have any paint left over? How much? Could money be saved if paint were purchased in smaller quantities? To find out, student will need to ascertain the price of different sized units of paint.

5-51 Build a City Skyline.

Materials: Ruler, scissors, graph paper, mural paper, tape, compass, pins, spray paint.

Purpose: Linear measurement, scale drawing, ratio, fractions, angles, geometric shapes.

Procedure: Student will make scale drawings. (Recommended scale 1 inch= 10 feet) for the following buildings:

a) Department store - 300 feet high, 500 feet wide.

b) Office building - 1,050 feet high, 250 feet wide.

c) Church building - 275 feet high*, 225 wide.
*a steeple makes the church 850 feet tall.

d) Apartment building - 725 feet high, 150 feet wide. Add a penthouse that is 25 feet high, 100 feet wide.

e) Convention center - 400 feet high, 300 feet wide. Semicircular dome 300 feet in diameter.

Graph paper may be used for making scale models. For effective displays, pin, tape or paste models onto mural paper. A more ambitious project can be done by creating a three-dimensional "city."

5-52 The Clip Shop.

Materials: Postal scale, box of paper clips of identical size.

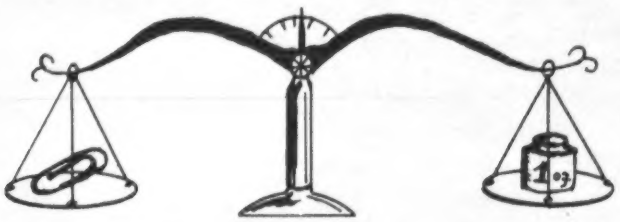
Purpose: Weight measurement, common and decimal fractions, numeration, division.

Procedure: The student is asked to find a way to get the weight of one paper clip. The following clues may be offered, as needed:

Clue #1 - How many clips are needed to make the scale read one ounce?

Clue #2 - If ___ clips weigh one ounce, how much would half that number of clips weigh?

Variations: Find the weight of: a single index card; one grain of rice, one kernel of corn; one thumbtack; one inch of yarn; etc.



5-53 In the Thick of Things.

Materials: A stack of 5"x8" cards, ruler with 1/4" calibrations.

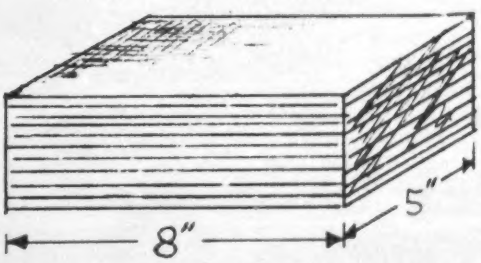
Purpose: Linear measurement, common and decimal fractions.

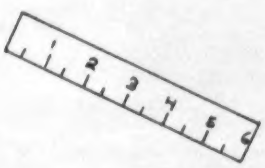
Procedure: The student is asked to find a way to measure the thickness of one card. The following clues may be offered as needed:

Clue #1 - Take a big stack of cards. Measure the height of the stack.

Clue #2 - Make a stack of cards one inch in height. Count the number of cards in the stack.

Clue #3 - If many identical notepads are placed in a stack one inch high, and you are told there are 100 notepads, how thick must each notepad be?





5-54 The Big Bean Contest.

Materials: Transparent container (with lid) filled with beans.

Purpose: Estimation, capacity, multiplication, data analysis range, mean (average), median, mode.

Procedure:

a) Ask pupils to estimate the number of beans in the container and to submit this estimate to the teacher. These estimates should be organized to give the statistical information about the range, mean, median, and mode.

b) Ask pupils to improve their estimates by using any lab materials they think would help them. You might hint that graph paper, measuring cups, balance scales, and rulers are available. An actual count of the beans, however, should not be permitted for estimation purposes.

c) Compare the second series of estimates with the first. Get an actual count to determine the reliability of the second series of estimates.

5-55 1,000,000 Paces.

Materials: New York State road map, map measurer.

Purpose: Linear measurement, map reading, mileage scales, ratio, unit conversions, division, numeration.

Procedure: Consider this problem first:

a) A man takes a walking trip. His pace is 36 inches. He walks 1,000,000 paces. How far is that in inches? Express this distance in feet and in miles.

b) Use the map measurer and a road map. Find a city which is 1,000,000 paces from Albany.

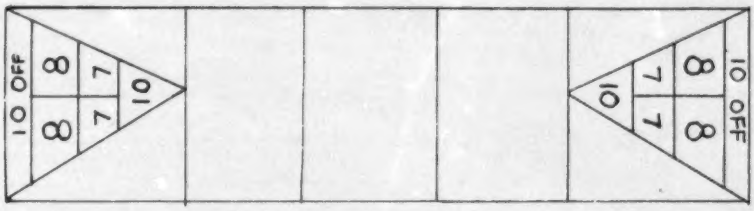
c) Metric measures can also be used for this experiment.

5-56 Make a Shuffleboard Court and Play.

Materials: Encyclopedia or game rules book, graph paper, tape, ruler, felt pen, tag board, checkers.

Purpose: Linear and angle measurement, scale drawing, ration, directed number addition, research.

Procedure: Have student obtain dimensions of a shuffleboard court and rules for playing and scoring. On graph paper or tag board, students should make miniature courts, drawn to scale. Instead of the usual full scale discs, students could play with checkers. The checkers can be moved by snapping the middle finger off the thumb against the checker. By devising nonstandard numbering of the court, you can provide computational practice with ordinary or decimal fractions.



Note: A full-scale shuffleboard court painted in the schoolyard or gymnasium can provide mathematics reinforcement and recreation during recess time.

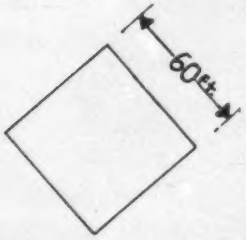
5-57 How Big Is a Big Baseball Diamond?

Materials: Encyclopedia or official game rule book, ruler, graph paper.

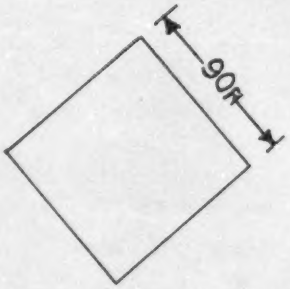
Purpose: Linear and area measurement, angles, ratio, scale drawings.

Procedure: Have students find out the dimensions of baseball diamonds. Compare the sizes of professional, senior league, little league, and softball diamonds. Make scale drawings or models of the different diamonds and the playing equipment.

Baseball



Little League Diamond



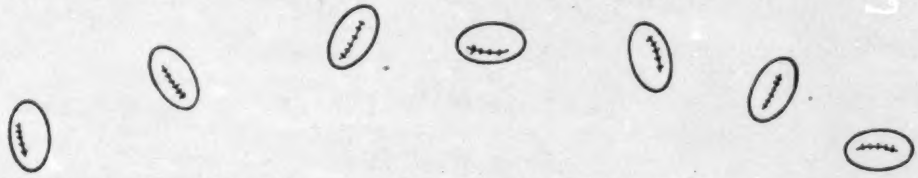
Major League Diamond

5-58 Get a Kick Out of It! (Activity for two students)

Materials: Open field, football, tape measure, chalk, football tee.

Purpose: Linear measurement, averaging, estimation, angles.

Procedure: Have the student draw two chalk rings around the ball, dividing it into thirds. Stand ball at an angle in the tee, laces up. Have the student draw a side view of the ball and attempt to predict paths of the ball if it is kicked in the top, middle, or bottom section. Sketch the predicted path for each kick. The student should then kick the ball five times for each position, and measure the distance the ball goes for each of the kicks. His partner should note the path and compare it to the predicted path. The average of the distance for five kicks should be computed for each position. The angle at which ball is placed in the tee is another variable which might be explored.



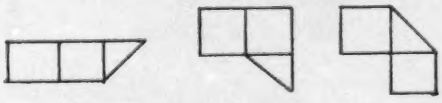
5-59 Game: Try for 50. (two to four players)

Materials: Graph paper (1" squares), scissors, 1" wooden cube to be marked for die, felt pen, oak tag, rubber cement.

Purpose: Area measurement, fraction addition.

Procedure: a) Have students make the parts for the game. Each face of a cube should be marked with a different numeral from a set, such as 1/2, 1, 2, 2 1/2, 3, 4. Shapes to represent values on the die should be cut from graph paper and mounted on oaktag. Variations of shape, when possible, for same value are desirable.

e.g. for 2 1/2 units -



b) Ten pieces for each value should be made.

The Game: Each player rolls the die in turn and takes a shape (from collection) which is equivalent to the value of the throw. Each player keeps the shapes he has rolled for. The first player to get an area of 50 square units is the winner. Penalties may be assigned if the player picks up an incorrect shape. Exchanges of equivalent pieces can offer further variations.

5-60 Gamesmanship.

Materials: Chessboard and men.

Purpose: Fractions, ratios.

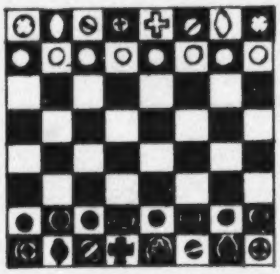
Procedure: Have pupils set up chessboard as if ready for a game. Many questions concerning ratios and fractions can be asked, such as:

a) What fraction of your men are pawns, castles, knights?

b) What is the ratio of castles to pawns?

c) What is the smallest fraction of all the men that could be left on the board at any time in the game?

d) What fraction of the board has no men? Have pupils make up additional questions.



5-61 Weight and Place in Football.

Materials: Almanac, official games rulebook, data for local football team.

Purpose: Measurement, comparison, averaging, data collection and tabulation.

Procedure: Have students collect data on the weight of each member of the local football team, and information as to position each man plays. Have students organize data so that each player is listed as to weight and position.

The average weight of team can be computed and compared to the average weight of the five heaviest men.

Have students sketch positions of men in a standard defensive formation and in a standard offensive formation, noting relationship of weight to location in each formation.

Have students compare weights of team with weights of a professional team.

Weights may be recorded in pounds and/or kilograms.

5-62 Is the Ball a High Bouncer?

Materials: Rubber ball, measuring tape, masking tape, graph paper (1/4"

Purpose: Linear measurement generalizations, graphing, estimation, data tabulation, averaging.

Procedure: Students are to be directed as follows:

a) Secure a measuring tape vertically against a wall, with the zero end at the floor.

b) Next to the measuring tape, drop a ball from a height of 60 inches. Observe the height of the ball's first bounce. Repeat this step at least four times. (Compute the average, too.)

c) Keep a record in a table like this:

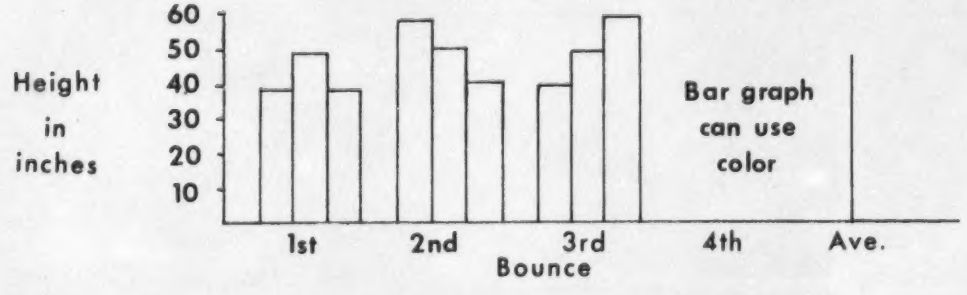
Ball dropped from 60 inches					
Experiment #	1	2	3	4	5
The ball bounced up to	___in.	___in.	___in.	___in.	___in.

d) Make another table of information as above, but drop ball from 36 inches this time. Do this four times. Compute the average.

Ball dropped from 36 inches					
Experiment #	1	2	3	4	Average
The ball bounced up to	__in.	__in.	__ in.	__in.	__ in.

e) Estimate, on basis of above findings, how high the ball might bounce from 48 inches. Now try it out, and compute the average.

f) Make a graph of all the data obtained. What do you notice?



5-63 What's the Angle?

Materials: Three different sized marbles, strip of metal foil about 4" x12", tape, chalk.

Purpose: Linear measurement, generalizations, graphing angles, estimation, data tabulation.

Procedure: Have students make a model of a ski jump by using a strip of foil to make a trough. The strip should be bent into a curve, and positioned as an inclined surface. The students can then roll the smallest marble down the trough and beyond. The distance that the marble rolls should be measured. This procedure should be repeated several times before an average is computed. The same process should be used for the next sized marble. Pupils should then estimate the average rolling distance for the largest marble before verifying this distance experimentally. The height, length, and shape of the ski jump may be varied to see what effects these have on rolling distance. Findings may be tabulated and graphed.

5-64 Broad Jump Fraction.

Purpose: Linear measurement, fractions, ratio, research data tabulation, graphing.

Procedure: Have student look up Olympic record for longest broad jump. Have him mark this distance on the playground or gym floor. Have students jump. Measure and record the distances. What fraction of the Olympic record is each jump?

They can make other comparisons, e.g., height or weight of pupils related to their jumping distances.

A graph summary can be made of the jumps of each class member.